

PORTFOLIO OPTIMIZATION

*Why preference-based optimization is the next generation
of portfolio construction methodologies*

There exist many methodologies to determine an optimal portfolio allocation on the market. In this document we list the most popular methods and compare them with a risk-preference based approach. We also provide a practical example including a back-test.

The use of mathematical methods in order to find an optimal weighting of assets in a portfolio traces back to more than a half century ago. In 1952, H. Markowitz published a ground-breaking paper on the optimization of portfolios based on a simple trade-off between risk (variance) and return (mean) given the correlation between the assets in the portfolio. Markowitz's *Modern Portfolio Theory* nowadays still represents the basis of education in finance. The resulting traditional portfolio allocation technique is often called "mean-variance" optimization, referring to the use of the first two moments characterizing the distribution of returns.

Ever since, numerous academics and experts have built further on this theory in order to circumvent some practical issues regarding the assumptions of the theory. Examples include:

- the integration of time-varying volatilities and correlations (the family of GARCH models introduced by Bollerslev and Engle)
- the inclusion of market views to circumvent the problem of expected return estimation (Black-Litterman)

Despite the progress made with these improvements, one of the major flaws of these methodologies is that risk remains narrowly defined as volatility. However, by definition, volatility is a symmetric, "one size fits all" risk measure, that does not take into account the behaviour of stock returns during turbulent market conditions. Recent market data has shown that the higher moments of the return distribution – i.e. the coefficients of asymmetry (skewness) and fat-tailedness (kurtosis) of financial asset returns – tend to correlate strongly when crises approach. Recent studies provide conclusive evidence that systematic skewness and kurtosis risk are in fact priced on the market. (Lambert and Hübner, 2013)

Furthermore, from an investor's perspective, another weakness of the mean-variance framework is that it does not take into account specific risk aversion and risk perception of an individual investor. However, we know that each investor has a different preference structure towards her investments. Traditional quadratic utility functions are too simplistic to reflect this heterogeneity. A proper, client-centric portfolio construction process has to reflect this complexity. This is the real challenge of portfolio optimization in the 21st century.

UTILITY FUNCTION

To tackle the shortcomings of traditional approaches that deny investors' sources of heterogeneity, the use of non-standard utility functions are necessary. In financial decision making, a utility function allows the investor to rank the expected wealth levels that result from alternative investment opportunities. In fact, unlike other theories, which are based on approximations, the expected utility concept is the only theoretical framework that takes actual attitudes and behavior into account.

To better understand this, one can usefully go back to the key assumptions underlying the concept. If a decision maker:

- prefers more money to less;
- respects a small set of standard rationality assumptions underlying Von Neumann and Morgenstern's expected utility theory;
- shows some risk aversion at all wealth levels; and
- has a consistent assessment of risk, i.e. always ranks risky assets independently of her portfolio composition.

then most traditional functional forms (quadratic, power, negative exponential, logarithm) fail, mostly because of the last criterion. To

include this criterion, the only appropriate utility function is the linear-exponential (*linex*) form, also called Bell's utility function, which is commonly written as

$$U(W) = W - be^{-cW}$$

where parameters "b" and "c" are positive investor-specific coefficients. Investors with high values of c put more emphasis on the possibility of capital loss while parameter b controls for their degree of risk aversion. In other words, risk preferences span on two dimensions: people do not only have preferences in terms of the quantity of risk taken (*risk aversion*), but also in terms of the type of risk (risk definition, sometimes associated to the behavioral finance concept of *loss aversion*). Bell's utility function faithfully integrates both dimensions of risk/investment preferences.

More specifically, the risk definition of an investment in a financial asset *only* depends on the value of parameter c (type of risk), which is just as investor specific as risk aversion parameter b is. Because it accounts for the two dimensions of the investor's intrinsic risk profile, Bell's utility function reflects individual preferences more properly than other (single parameter) utility functions. Any market-wide definition of risk, like volatility, can only result from an aggregation of identical risk measures. By acknowledging the diversity of investors, Bell's utility function does not force a common measure to be shared by everyone. Thus, using this framework, the advisor or asset manager can discriminate between risk measurements on the one hand and the trade-off between risk and return on the other hand.

In a nutshell, any "one size fits all" risk measure like volatility may be reasonable at the market level, but is doomed to be incorrect at the individual level.

Appropriateness for extreme risks

In addition to reflecting investors' heterogeneity, Bell's utility function is well suited to deal with financial assets presenting a non-normal distribution (as most assets do).

Whatever the allocation system that is put in place, it is necessary to implement a convincing way of tackling extreme risks with metrics other than the volatility, even when conditional models are used. The low volatility of some assets or classes of assets is mostly illusory. Think of the very low volatility of Equity Market Neutral hedge funds, their extreme risks during the financial crisis were huge. Clients with a strong emphasis on capital protection were bound to be dissatisfied by these kinds of products.

Contrary to traditional linear utility functions, the Bell utility function is designed to integrate the first four moments of any distribution, i.e. the mean and variance, but also the skewness (i.e. the asymmetry of returns) and the kurtosis (i.e. the tail risk in returns).

Reactivity

There is another important advantage of integrating the concept of non-normality into the analysis. Higher moments of the return distribution provide a really proactive way to anticipate market crises. This property results from the speed at which correlations increase in the tails of returns: they grow much faster than covariance when the market is on the verge of a crisis.

Incorporating higher moments into the risk function and matching them with expected returns, provides a true dynamism in the allocation. Models solely based on volatility are thus reductive, and as a consequence miss the same level of reactivity and capital protection.

OTHER METHODOLOGIES

Risk Parity

The Risk Parity methodology is popular since it alleviates the delicate problem of expected return estimation by largely avoiding it, and has proven quite successful over the past decades. The idea is that portfolios should focus on risk allocation (often defined as volatility) rather than on capital allocation amongst different asset classes to enhance the diversification of the portfolio. In other words, rather than

allocating weights to asset classes, the methodology states that each asset class should contribute the same amount of risk to the portfolio.

This methodology has however many flaws.

- The measure of risk, typically volatility, is unique for all investors: it does not tell the complete story about loss aversion and the heterogeneity of investors;
- The model generates leveraged portfolios, favoring fixed income products, mainly driven by the previous point (volatility as risk parameter). Declining interest rates, as observed during recent years, are very favorable for the often promising back-tests, but this begs the question whether interest rates can keep declining forever at current pace. Furthermore, the model transfers volatility risk into other types of risk (like leverage);
- By avoiding the risk-return tradeoff, this approach is incompatible with expected utility maximization, and can thus not achieve any provable coherence with investor profiles. In particular, risk parity portfolios cannot be MiFID-compliant.

Bottom line: the approach heavily hinges on the presence of a significant term premium (i.e. the difference between long-term and short-term bonds), while Markowitz aims at generating return by exposures to market risk. (The difference between stocks and bonds). There is *a priori* no dominance of one against the other, but at least both approaches clearly involve a high-return/high-risk trade-off. Only the nature of risks differs, but there is no theoretical ground for saying that term risk is better remunerated than market risk.

One of the key characteristics of risk parity is that it avoids the estimation of expected returns. According to us, the inclusion of expected returns in the optimization framework is necessary and useful. It is necessary because – let's face it – the decision to allocate resources to one asset rather than another is mainly a matter of return. One should not resign the responsibility of trying to assess potential future returns simply because this is difficult. And, no one can reasonably try to infer a distribution of returns without the expectation.

Including expected returns in the asset allocation process is also useful, because it introduces relevant

information. We have expertise, both from scientific research (estimating expected returns using EWMA for example, to give the latest returns more weight in the analysis) and from practice (using the asset manager's views regarding expected returns rather than calculated numbers) that enables us to improve the mere measurement of risks with an estimation of what returns could be.

Scenario approaches

At the other end of the spectrum of portfolio optimization techniques, scenario approaches are sophisticated engines that aim to represent the real world in the most comprehensive manner.

One of the main advantages of scenario based optimization (based on simulation methods) is the possibility to integrate market behavior that goes beyond mean-variance. This numerical approach maps financial securities onto risk factors whose multivariate distribution yields randomized drawings. Several reduction techniques make the procedure, which is presumably very computer intensive, workable within a reasonable timeframe.

This forward-looking framework is very appealing in the context of dedicated portfolio management solutions. It can integrate non-linear products like bonds and options up to the most complex structured product. Thanks to a potentially unlimited number of simulations, tail-based risk measures can be computed and used as risk minimization criteria.

Promoters of these approaches usually exhibit a preference for the Conditional Value-at-Risk (CVaR), also called Expected Shortfall. This risk measure is a "coherent" one, unlike the variance, and is thus a defensible proxy for extreme risks.

Nevertheless, there are also non-negligible drawbacks from such methods. First, they are complex, parametric, and ultimately look like a pure "black box". The output of a scenario system is thus hardly recognizable from the hypotheses. This makes them prone to the "GIGO" syndrome ("garbage in, garbage out"). Next, the use of scenarios is mostly warranted in order to use a tail risk estimator like the CVaR – for more traditional risk measures, one does not need this complex machinery. But there is no investor, besides institutional ones, whose preferences are consistent with such risk

measures! Thus, although this approach could be acceptable when setting risk budgets to control portfolio risk, this is not a suitable methodology for individualized, profiled portfolios matching investors' expectations for risk and return.

Finally, nothing guarantees that scenario analysis could provide meaningful portfolio decompositions. The framework belongs to the class of full valuation models, which lacks the analytical simplicity that allows fully understanding the contribution of each portfolio building block to the overall investor utility.

PRACTICAL EXAMPLE

Optimization using ETF's

After comparing the utility based optimization to other methodologies, and pointing out the pros and cons, let us look at a practical example.

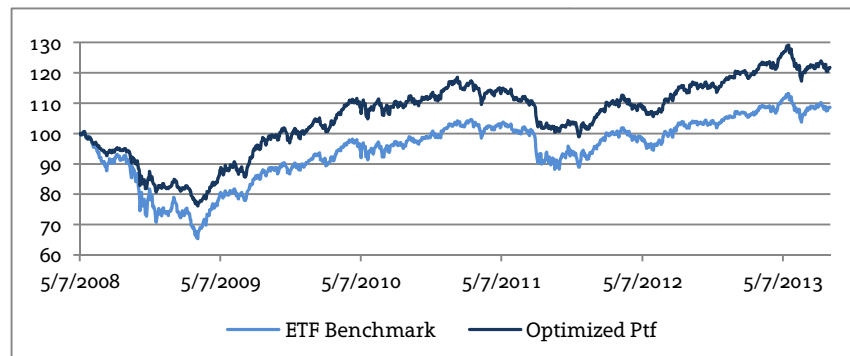
We developed a (out-of-sample) back-test based on 27 directly investible ETF's dispersed over various asset classes ranging from Equity (EU and World), Bonds (Government, Corporate HY and IG), Private Equity, Real Estate and commodities. Each ETF has a minimum allocation of 1.5% in the portfolio and can build up to 6%. The rest amount is invested in cash. The equity part is allowed to build up to 70%, the cash part has a maximum weight of 60%. The risk profile used in this example corresponds to a dynamic-protective (low b, high c) investor.

To evaluate the quality of the optimization, we constructed a benchmark using the same underlying ETF's as the optimization. In order to match the risk exposure of our optimized portfolio, we calculated the average weight per asset class over the entire back-test (Equity, Bonds, Cash) and distributed this exposure evenly over the different ETF's. This resulted in a benchmark with 22.68% invested in the cash index, 63.88% in Equities and 13.44% in Bonds.

Results

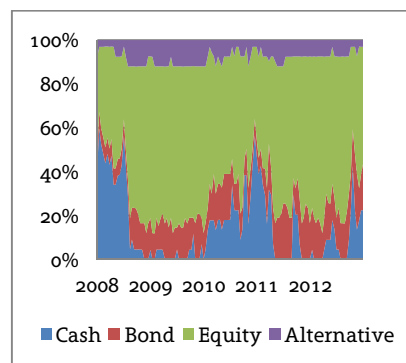
With rebalanced compositions every two weeks, we reconstructed the historical performance of the funds. The result of the back-test over 5 years can be found in the line chart.

When looking at the price history since May 2008 (65 months), we notice a clear capital protection in 2008 and a cumulated outperformance of more than 10% over the entire period (2008-2013). The average yearly return amounts to



5.68%, while the benchmark only reaches 4.51%.

Also, the portfolio risk measures are significantly lower than the benchmark: the volatility shrinks to 10.7%, almost 3% lower than the benchmark. Consequently, the Sharpe ratio exceeds 53%, a much (and significantly) higher figure than the 34% of the benchmark. Furthermore, parameters going beyond volatility, like kurtosis (1.69 compared to 4.8 for the benchmark) and maximum drawdown, are in line with the concepts of the utility based model: protecting capital during periods of market stress.



The last graph shows the asset allocation of the optimized fund during the 5 years of back-test. We clearly see that the model shifts its allocation towards cash and bonds during periods of market stress, while investing at maximum in high return equity during upturns. The turnover over the entire period is a reasonable 2.47 per year.

Using a similar approach we have developed a methodology to reverse-profile a portfolio –i.e. to determine the risk profile of the investors for which it is suitable. This is the subject of an upcoming publication.

ABOUT GAMBIT

GAMBIT Financial Solutions SA ("Gambit") was created in 2007 as a university spin-off from HEC Management School of the University of Liège (HEC-ULg). While keeping a strong partnership with HEC, it has since evolved into a full-fledged commercial company with a strong international focus.

The company delivers to its customers a complete suite of interactive, client-centric decision-making support solutions in investment advisory, portfolio and funds management, risk and compliance monitoring.

We also provide advisory services in portfolio optimization, risk monitoring, and funds assessment.

Our customers include Retail Banks, Private Banks, Assets Managers, Insurance companies, Risk Managers, Family Offices and Fund Managers.

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