# The Simple Rules of Complex Networks
## A Heuristic for Determining the Potential Complexity of Any Network and Making Structural Predictions

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**Abstract: Abstract: Network theory has broad application in the physical, natural, and social sciences. The study of complex networks, and their applications to the study of complex systems, have focused predominantly on: (1) the elemental vertex-edge structure (a.k.a., nodes and relationships), and subsequently (2) the dynamics that occur as a result of this basic structure (e.g., diameter, distribution, small world, contagion, etc). This network theory methodology has provided powerful quantitative tools in the interdisciplinary study of complex systems, and has enriched our thinking about them. However, the simplifying assumptions of the network theory framework have also informed the field of systems thinking, sometimes to its detriment. Network-thinking tends to shoe-horn a number of important elemental structures of complex systems into node-edge relational structure. By assigning these elemental structures to edges, both elemental and emergent complexity can be lost. This paper articulates how DSRP Theory can enrich network thinking about complex systems by: (1) identifying the elemental structures that are typically hidden in network models, (2) quantifying their nature and abundance, and (3) explicating their potential contribution to the intrinsic function and emergent complexity of systems. Specifically, we detail several DSRP heuristics for determining how many elements potentially exist in any network model, demonstrating the effectiveness of DSRP as a "universal cognitive grammar" for identifying and analyzing the structural potentials in complex systems.**

DSRP | systems thinking | complex networks | counts | network theory | structural predictions

## Contents



## 1. Systems Thinking and Network Thinking

It is well established that Systems Thinking (ST) is a complex and adaptive phenomena borne of a universal set of simple rules(1, 2). Networks are a powerful visualization tool for analyzing and understanding systems. In that sense, *network thinking* and *systems thinking* are synonymous because the terms *networks* and *systems* are both abstract and general terms used to describe any phenomena with two or more related elements.

## 2. The Bridges of Konigsberg

Leonid Euler (1735) famously solved the Konigsberg Bridges problem and in the process invented network theory (also known as graph theory). Since then, networks have provided a simple, abstract representation of both simple and complex systems (3, 4) and proven to be an invaluable interdisciplinary tool that is ubiquitously used in every discipline and in every sector. The Konigsberg Bridges problem was to determine whether a person could walk through the city and cross each bridge once and only once (5). Euler's great insight was the use of abstraction, to reduce the four land-masses to nodes (vertices) and the relationships (edges) (See Figure 1).

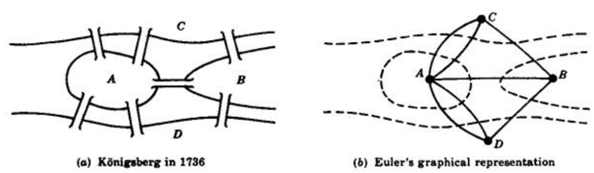


**Fig. 1.** The Konigsberg Bridges that inspired Network Theory

Thus, Euler showed us that networks were based on two *elemental* structures: the vertex and the edge, which are also commonly referred to as nodes and links, or nodes and relationships (See Figure 2). This simple, elemental structure has proven invaluable for discovering and understanding all kinds of larger-scale behavior of networks (e.g., diameter, distribution, connection patterns, small world effects, emergence, robustness, adaptivity, contagion, etc.)

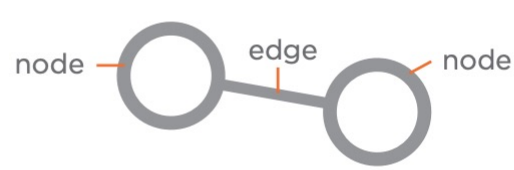


**Fig. 2.** Elements of Network Theory

Citation information: Cabrera, D., Arnold, L., and Cabrera, L. (2019). The Simple Rules of Complex Networks: A Heuristic for Determining the Potential Complexity of Any Network and Making Structural Predictions. *Journal of Applied Systems Thinking* (20) 5.

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There are however, numerous other elemental structures that exist in networks. These structures are explicated by DSRP Theory. DSRP Theory provides a number of important variables that are universal to systems abstractly, and specifically to networked phenomena. Like Network Theory, DSRP has a very basic structure. And, like Network Theory, the simplicity of this structure belies its potential complexity. The DSRP structure is given in Table 1.

**Table 1. Basic Structures**

| Patterns | Elements | | |
|---|---|---|---|
| Distinctions ($D$) | identity ($i$) | $\leftrightarrow$ | other ($o$) |
| Systems ($S$) | part ($p$) | $\leftrightarrow$ | whole ($w$) |
| Relationships ($R$) | action ($a$) | $\leftrightarrow$ | reaction ($r$) |
| Perspectives ($P$) | point ($\rho$) | $\leftrightarrow$ | view ($v$) |

DSRP Theory states that elemental pairs exist that make up each of the four simple rules of cognition: making Distinctions ($i$,$o$), organizing ideas into Systems ($p$,$w$), recognizing Relationships ($a$,$r$), and taking Perspectives ($\rho$,$v$). It can be expressed as a complex adaptive system or CAS, in which the agents are informational variables and the simple rules are DSRP patterns and their co-implying elemental base-pairs. In this way, DSRP is to the formation and evolution of cognition as ATCG is to the formation and evolution of biology (life).

Systems thinking is described as a continuous and recursive feedback loop borne of DSRP processing of information, as seen in Figure 3.

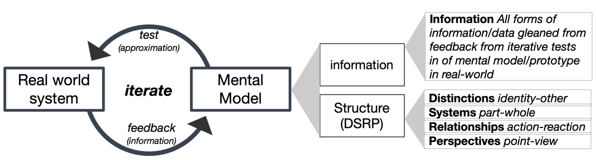



**Fig. 3.** The ST Loop

This recursive loop is described by a simplified equation explicated in Table 2.

**Table 2. The equation of Mental Models ($\mathbb{M} = \mathbb{I} \otimes \mathbb{T}$)**

| Mental Model ($\mathbb{M}$) | = | Information ($\mathbb{I}$) | Thinking ($\mathbb{T}$) |
|---|---|---|---|
| A mental model is synonymous with knowledge, meaning, construct, model, schema, idea, concept, etc. | | Information is synonymous with symbolic variables, content, data, labels, words, language, materials, etc. And, can also be understood as the fundamental function of the material world (e.g., to transport information) | Thinking refers to both noun-like structures (a thought) and verb-like processes (a thinking process). It is synonymous with encoding, organizing, or structuring content in order to give |

And further explained in the following equation which is explicated in Figure 4.

$$\mathbb{M}_n = \bigoplus_{\mathbb{I}} \bigotimes_{j \leq n} \mathbb{T}\left\{: D_o^i \circ S_w^p \circ R_r^a \circ P_v^\rho :\right\}_j$$

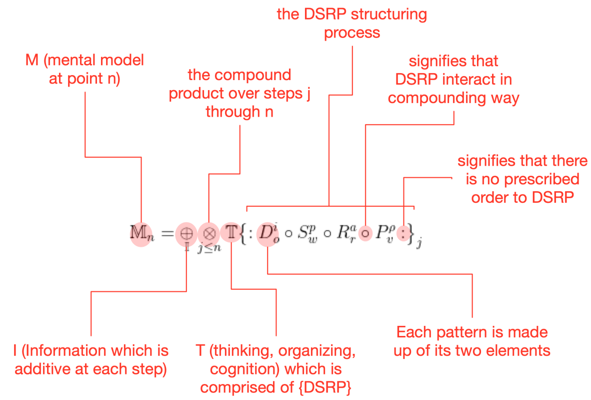



**Fig. 4.** Expanded Explanation of Equation

Table 3 summarizes some of the main differences between the basic structures of network theory and that of DSRP. At its core however, DSRP is an extension of network theory.

**Table 3. Net Difference Basic Structures**

| Network Theory | DSRP Theory |
|---|---|
| 1. nodes defined<br>2. edges defined as connections between nodes | 1. nodes defined (called identities) (no difference)<br>2. other (nodes) co-define identities<br>3. edges defined (called Relationships) (no difference)<br>4. defines action-reaction structure of Relationships<br>5. any edge can become a node<br>6. any node can be a whole (contain parts) or a part (can belong to another node); this includes edge-nodes<br>7. any node can be the point or the view of a Perspective |

The additional structures in DSRP Theory have many uses, but the predominant use is to make *structural predictions*. DSRP rules help us to make predictions about the structure our mental models or reality is capable of taking. This awareness of potential structure, allows us to identify gaps in our knowledge and identify where new knowledge could be discovered or created. For example, if you were a detective in a real-life game of CLUE and I tell you that there were 6 people at the party where the murder took place, it would be quite easy to count the number of relationships (or possible interactions) that are structurally possible (using the expression $n(n-1)$) simply based on $n$ equalling 6: $n(n-1) = 6(6-1) = 6(5) = 30$. So, there are 30 possible relationships among the party-goers. This of course does not tell you anything about the *reality* of interconnections. It may be true that Professor Plum and Miss Scarlet never had a conversation, but as a detective, it is your job to predict that they structurally *could have* and discover whether this relationship should be drawn or not drawn. In order to make structural predictions, it is often useful to develop an awareness of the counts associated with such structures. A heuristic for doing so aides this task. Heuristics are used to ascertain the maximum *degrees of freedom* or *potential complexity* of a system or network.

## 3. Relating Nodes (R)

What defines an edge? In network theory an edge is defined by the nodes it links. Thus, in the Konigsberg example in Figure 1, one of the edges might be defined as $\overline{AB}$. In Figure 5 you can see that the "Relationship Rule" or "R-Rule" makes it explicit that any node can (or cannot) be related to any other node. This includes (as we will later learn in the "S-Rule") nodes that are sub-parts of other nodes and it also utilizes the is/not structure—as we will later learn in the "D-Rule"—that a node *is* and/or *is-not* related to another node. This, too, is important, as both the structure and dynamics of systems (networks) are highly dependent on both what is and what is not related.

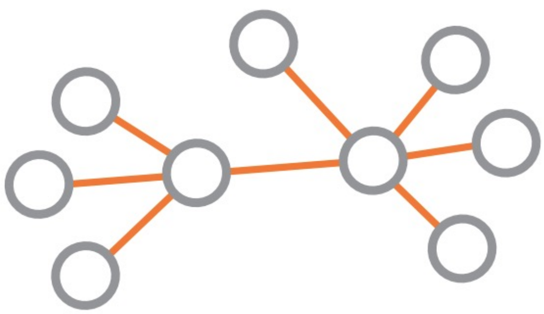


**Fig. 5.** Relationships in Networks

Calculating the number of relationships of a given network can not only make structural predictions about maximum number of possible relationships (R-Rule) but also the maximum number of possible relationships that should or *could* be distinguished (D-Rule). In Figure 6, for example, all of the nodes that exist could be related, but only 8 relationships exist (or 16 if one counts each as a two-way relationship). Using the basic heuristic $n(n-1)$ we can see that for $n = 9$ (the original "root" set of nodes) the number of possible relationships among them is $n(n-1) = 9(9-1) = 9(8) = 72$. Yet, in Figure 6 only 8 relationships have been identified as being salient. So while there are 72 possible relationships and 36 possible relational-distinctions ("RDs" or edges with nodes on them), there are only 8 identified (or 16 if one counts each as a two-way relationship) and there are therefore 56 potential two-way relationships where a structural prediction can be made. By structural prediction we mean that one might say, "these are all the structural possibilities, but which one's are salient to our particular analysis?" In other words, additional unseen Relationships exist that can be predictably identified.

A simple counting of the elements in the Konigsberg network example (Table **??**) shows the four nodes (A,B,C, and D) and the 7 edges illustrated in *b* Figure 1 in the first two columns.

**Fig. 6.** Relationship-Distinctions (RD) in Networks

**Table 4. Various Ways to Count Rs between *n* identities**

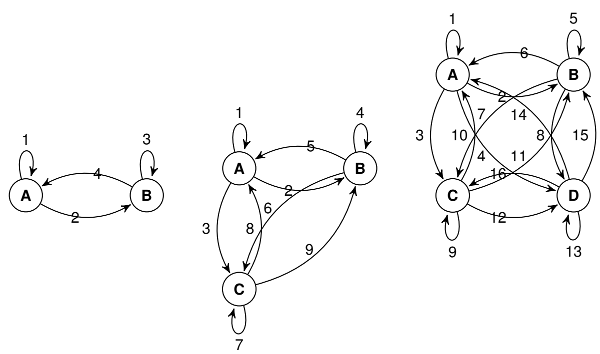
| Nodes $n=4$ | Simple R $n(n-1)/2$ | 2-way $n(n-1)$ | 2-way R+ self $n^2$ | Complex $R_r^a$s $2(n^2)$ |
|---|---|---|---|---|
| A | $\overline{AB}$ | $\overline{AB}$ | $\overline{AA}$ | $\overrightarrow{A^d}$ $\overleftarrow{A_r}$ |
| B | $\overline{AC}$ | $\overline{AC}$ | $\overline{AB}$ | $\overrightarrow{A^d}$ $\overleftarrow{B_r}$ |
| C | $\overline{AD}$ | $\overline{AD}$ | $\overline{AC}$ | $\overrightarrow{A^d}$ $\overleftarrow{C_r}$ |
| D | $\overline{BC}$ | $\overline{BA}$ | $\overline{AD}$ | $\overrightarrow{A^d}$ $\overleftarrow{D_r}$ |
| | $\overline{BD}$ | $\overline{BC}$ | $\overline{BB}$ | $\overrightarrow{B^d}$ $\overleftarrow{B_r}$ |
| | $\overline{CD}$ | $\overline{BD}$ | $\overline{BA}$ | $\overrightarrow{B^d}$ $\overleftarrow{A_r}$ |
| | | $\overline{CA}$ | $\overline{BC}$ | $\overrightarrow{B^d}$ $\overleftarrow{C_r}$ |
| | | $\overline{CB}$ | $\overline{BD}$ | $\overrightarrow{B^d}$ $\overleftarrow{D_r}$ |
| | | $\overline{CD}$ | $\overline{CC}$ | $\overrightarrow{C^d}$ $\overleftarrow{C_r}$ |
| | | $\overline{DA}$ | $\overline{CA}$ | $\overrightarrow{C^d}$ $\overleftarrow{A_r}$ |
| | | $\overline{DB}$ | $\overline{CB}$ | $\overrightarrow{C^d}$ $\overleftarrow{B_r}$ |
| | | $\overline{DC}$ | $\overline{CD}$ | $\overrightarrow{C^d}$ $\overleftarrow{D_r}$ |
| | | | $\overline{DD}$ | $\overrightarrow{D^d}$ $\overleftarrow{D_r}$ |
| | | | $\overline{DA}$ | $\overrightarrow{D^d}$ $\overleftarrow{A_r}$ |
| | | | $\overline{DB}$ | $\overrightarrow{D^d}$ $\overleftarrow{B_r}$ |
| | | | $\overline{DC}$ | $\overrightarrow{D^d}$ $\overleftarrow{C_r}$ |
| 4 | 6 | 12 | 16 | 32 |

This is how the actual bridges in Konigsberg were (i.e.,where $\overline{AB} = \overline{BA}$). So, the total possible bridges (or edges/relationships) we *could* have for this network of 4 nodes could be quickly counted using the formula $n(n-1)/2 = 4(4-1)/2 = 4(3)/2 = 6$. This formula works reasonably well to count the basic number of connections between $n$ nodes but it treats $\overline{AB} = \overline{BA}$. In other words, it treats a drive from New Jersey to New York as the same as a drive from New York to New Jersey.

Using the base formula $(n(n-1))$ we can see that it works based on the idea that in any network of $n$ nodes, the number of nodes that each node can connect to is one less than the total number of nodes. Ergo, when the number of nodes is 4 as in the Konigsberg example $(n = 4)$, each node can relate to $n(n-1)$ nodes, or $4(4-1)$ or $4(3)$. The $n(n-1)$ formula counts $\overline{AB}$ and $\overline{BA}$ as not equal $(\overline{AB} \neq \overline{BA})$. The third column in Table 4 shows all of the relationships using this formula equal to 12.

But, if the desire is to identify *all* of the *degrees of freedom* in a network of $n$ nodes, then we have left out the node's ability to relate to itself. This self-relationship, in some systems (particularly psychological or sociological networks), is critically important. For example, we know that bias plays a significant role in individual behavior. Bias, in turn, can be mitigated by metacognition (awareness of one's tendency toward certain faulty mental models). Metacognition, awareness, bias recognition, etc. are all self-relationships that can effect the behavior of the node and therefore effect the emergent behavior of the network itself. Thus, for a network where $n = 4$, the fourth column of Table 4 illustrates that there are 16 possible unique relationships (including self-relationships) which is further shown in the network images in Table 5.

**Table 5.** $n^2$ **counts for n= 2, 3, and 4**




$n = 2$, $edges = 4$ $n = 3$, $edges = 9$ $n = 4$, $edges = 16$

Thus—*when including self-relations*—the formula to identify the total degrees of freedom in terms of Relationships for $n$ nodes is not $n(n-1)/2$, nor $n(n-1)$, but $n^2$. The Relationship Rule or "R-rule" in DSRP Theory states that *relationships are universally structured as co-implying elements: action (a) and reaction (r).* Therefore, if one wants to account for the action and reaction variables of all possible Relationships between n nodes, including self-relations, the equation must be $2n^2$, where each directional relationship is not merely a single variable but two: the action of A on B and the reaction of B for the relationship $\overline{AB}$ and vice versa for $\overline{BA}$. Thus, when we want to calculate the maximum degrees of freedom regarding Relationships in a network of $n$ we use the formula $2n^2$ as seen in column 5 of Table 4. Thus, the formula $2n^2$ provides the maximum number of Rs (or degrees of freedom) in any given network of size $n$ and column 5 lists all the specific relational variables for the $ABCD$ network (i.e., 32).

In addition to the counting of relational degrees of freedom, R-Rule can be mixed with D-Rule such that every one of the $2n^2$-relationships can become a new node, thus increasing the original "root" $n$ of the network.

### Heuristic to Determine Potential Complexity of R

The potential complexity of R at an arbitrary root stage is defined as the maximum number of actions and reactions that can be formed from $n$ root identities, without compounding with D, S, or P. The first count does not include "self-relations," which are only sometimes useful in conceptual models. The correction to the results when self-relations are also included.

When "self-relations" are not counted, the case of one root identity is trivial and has zero degrees of freedom. For the case of two root identities $\{1\ 2\}$, each can be regarded as either an action or reaction of a relationship $1 \leftrightarrow 2$ (i.e. $1 \xrightarrow{a\ \ r} 2$ and $1 \xleftarrow{r\ \ a} 2$). These possible relationships can be enumerated by pairing them with the orderings or permutations of the two elements, i.e. $1 \xrightarrow{a\ \ r} 2 \ \cong\ 1 \rightarrow 2$, and $1 \xleftarrow{r\ \ a} 2 \ \cong\ 2 \rightarrow 1$. In other words, there are two elements for each R, and each R is one of the two possible orderings of $\{1\ 2\}$, which gives $2 \times 2 = 4$ degrees of freedom. Notice that "self-relations" such as $1 \rightarrow 1$ are not included.

Since R is fundamentally bivalent, any arbitrarily complicated relationship among three or more identities can be viewed as the composition of relationships between pairs of identities. So partitioning the R-counting in terms of permutations of couples as above extends to any number of root identities. For example, in the case of three root identities $\{1\ 2\ 3\}$, we have the following R's:

$$\begin{array}{ccc} 1 \rightarrow 2 & 1 \rightarrow 3 & 2 \rightarrow 3 \\ 2 \rightarrow 1 & 3 \rightarrow 1 & 3 \rightarrow 2 \end{array}$$

which are counted by the number of permutations of two objects drawn from a set of three objects, given by $\dfrac{3!}{(3-2)!}$. Further, each of these R's contains two degrees of freedom, one $a$ and one $r$. So the total number of $a$'s and $r$'s is $2 \times \dfrac{3!}{(3-2)!} = 12$.

It should be clear now that for $n \geq 2$ root identities, the maximal number of $a$'s and $r$'s is twice the number of permutations of two objects drawn from a set of $n$ objects

$$\#a + \#r = 2 \times \frac{n!}{(n-2)!} = 2n\,(n-1)$$

Note that we have not counted relationships between subgroupings such as $1 \leftrightarrow \{2\ 3\}$, since the description of such relationships necessarily refers to compound DSRP structure. For example, within the root identity set $\{1\ 2\ 3\}$, the relationship $1 \rightarrow \{2\ 3\}$ first constructs $\{2\ 3\}$ as a whole and regards it as an identity before relating $\{1\}$ to it, which is a composition of R with S and D.

To correct these formulae for the inclusion of "self-relations" such as $1 \rightarrow 1$, one additional action and one additional reaction is added for each possible self-relationship. This gives an additional $2n$ elements, bringing the total potential complexity to $2n^2$. Note that for the trivial case of one root identity, this count gives two degrees of freedom, corresponding to the one action and one reaction of the self-relationship.

## 4. Systematizing Nodes (S)

The ways nodes are defined and related matters, but so too does the way they are grouped. In DSRP Theory, we call this *systematizing* and the Systems Rule or "S-Rule" provides that *systems are universally structured as co-implying elements: part (p) and reaction (w).* This means that any of the $n$ nodes in a network has the potential to be a part of a grouping (or several groupings) and also has the potential to be its own grouping (a whole) to which parts belong. In Figure 7 we revisit our emerging network of nodes to illustrate how each of the nodes (both the original 9 "root" nodes and the 7 of the 8 subsequent relational-nodes) can be further broken down into part-whole systems with equal or greater complexity to the hierarchical level up. Note that the we've selected an arbitrary number of parts for each whole of 3, but this number could be any number. Also note that the one relationship where we chose not to identify as an "RD" or relational-node, cannot be broken into parts (i.e., systematized) because there is no node on which to operate. But here again, the structural prediction can be made to let us know that there is cognitive and real-world possibility lurking there, that may or may not be salient and that may or may not be acted upon.

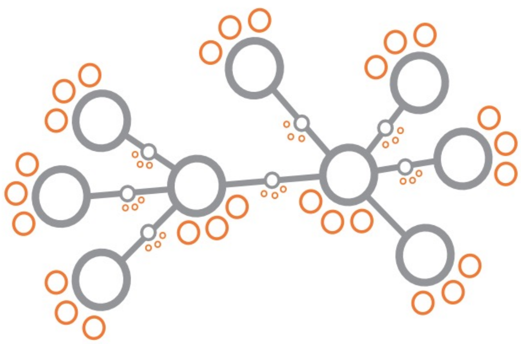


**Fig. 7.** Part-Whole Systems in Networks

In order to determine the maximum degrees of freedom in a network we will also want to be able to count the number of ways $n$ nodes can be organized into groups. This is given by the formula, $2^n - 1$. Simply put, this formula takes $n$ nodes and does the following in Figure 8:

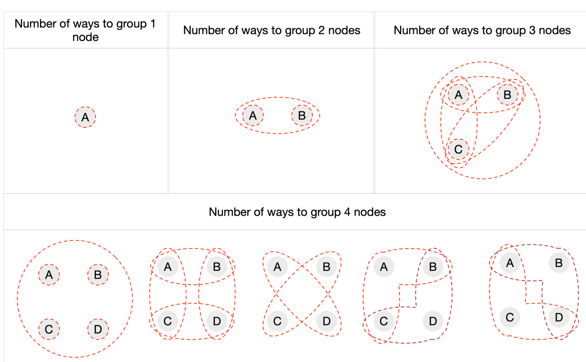



**Fig. 8.** S-Rule $2^n - 1$

First, it takes groups of 1 for $n$. So if $n = 10$ then there can be 10 groups made up of 1 node each. Next, it takes groups of 2 for $n$. Next, it takes groups of 3 for $n$. Next, it takes groups of $n$ for $n$, and so on.

In addition to groupings of $n$ nodes, DSRP's S-Rule provides for deconstructing any node into any number of parts, adding infinite dimensionality to the network. The number of parts (new nodes) that can be added to any existing node is effectively infinite, but in practical terms it is usually a number between 1 and 100. Thus, we can add the number of parts-added-per-node ($n_p$) to the original $n$.

## Heuristic to Determine Potential Complexity of S

The potential complexity of S at an arbitrary root stage is defined as the maximal total number of parts and wholes that can be formed given $n$ identities which define the root system.

As a simple example, consider the root system consisting of two identities {1 2}. It has parts {1} and {2}, and wholes {1}, {2}, and {1 2}. This gives a potential complexity of $2 + 3 = 5$ degrees of freedom. In principle one can also regard {1 2} as part of itself. In some conceptual models, this "self-part" structure may be useful to consider. So in this case, the counting would yield a potential complexity of $3 + 3 = 6$ degrees of freedom. However in many conceptual models, the "self-part" structure is not meaningful. Thus, the general potential counting does not include the "self-part" structure, but comment on the result when it is included is also offered.

DSRP describes how the identities denoted {1} and {2} can be broken further into part-wholes, ad infinitum. However, the next stage of part-whole structure requires the existence of new identities contained in {1} and {2}. This addition of identities can be accounted for as a subcase of the root stage with the appropriate number of identities. It is both consistent and convenient to partition the counting of part-whole degrees of freedom in terms of a fixed number of root identities.

To obtain the formula for the potential complexity of S as a function of the number of root identities, the counting of part-wholes is organized in the following:

| {1 2} | | p | w |
|---|---|---|---|
| | {1 2}: | {1}, {2} | {1}, {2}<br>{1 2} |

**#p:** $1 \times 2$ **#w:** 3

The number of parts by the subsystem of which they are to be regarded as a part. For example, the next largest root system has the following:

| {1 2 3} | | p | w |
|---|---|---|---|
| | {1 2 3}: | {1}, {2}, {3},<br>{1 2}, {1 3}, {2 3} | {1}, {2}, {3}<br>{1 2}, {1 3}, {2 3}<br>{1 2 3} |
| | {1 2}:<br>× 3 | {1}, {2} | |

**#p:** $1 \times 6 + 3 \times 2$ **#w:** 7

Note for example that regarding {1} as a part of {1 2} is distinct from regarding {1} as a part of {1 2 3}, and should be counted separately. However in this example, regarding {3} as a part of {1 2 3} is the same, regardless of whether its complementary parts are regarded as {1 2} or as {1} and {2}. So these two possibilities should not be counted as distinct in the basic part-count. Distinguishing them is a compound DSRP operation, such as describing the R or P structure of the S. The next largest root system is as follows:

| {1 2 3 4} | | p | w |
|---|---|---|---|
| | {1 2 3 4}: | {1}, {2}, {3}, {4} | {1}, {2}, {3}, {4} |
| | | {1 2}, {1 3}, {1 4}, {2 3}, {2 4}, {3 4} | {1 2}, {1 3}, {1 4}, {2 3}, {2 4}, {3 4} |
| | | {1 2 3}, {1 2 4}, {1 3 4}, {2 3 4} | {1 2 3}, {1 2 4}, {1 3 4}, {2 3 4} |
| | {1 2 3}: | {1}, {2}, {3} | {1 2 3 4} |
| | | {1 2}, {1 3}, {2 3} | |
| | $\times$ 4 | | |
| | {1 2} | {1}, {2} | |
| | $\times$ 6 | | |

**#p:** $1 \times 14 + 4 \times 6 + 6 \times 2$ **#w:** 15

Since each identity is apparently a part of several subgroupings as well as the root system, it is helpful to partition the part-count by subgroupings. It should be clear from the tables that the subgroupings partition the part-count binomially. More explicitly, the general formula for the number of parts of a system at step $n \geq 2$ is schematically:

$$\#p = \sum_{j=2}^{n} \text{(\#subgroupings of size j) x (\#subgroupings of subgroupings of size j) - 1}$$

$$= \sum_{j=2}^{n} \binom{n}{j} \left(2^j - 2\right)$$

and the number of wholes at step $n \geq 2$ is just the number of possible subgroupings of the root:

$$\#w = \#subgroupings$$
$$= 2^n - 1.$$

So the total potential complexity of S is the sum of the total maximal number of parts and wholes*:

$$\left|S_w^p\right| = \#w + \#p$$
$$= \left(2^n - 1\right) + \sum_{j=2}^{n} \binom{n}{j} \left(2^j - 2\right)$$
$$= \left(\left|\mathcal{P}\left(\{S_w^p\}_r\right)\right| - 1\right) + \sum_{j=2}^{n} \binom{n}{j} \left(\left|\mathcal{P}\left(\{S_w^p\}_j\right)\right| - 2\right)$$

whereby $\{S_w^p\}_r$ is the root system regarded as a set in traditional set theory, and $\mathcal{P}\left(\{S_w^p\}_r\right)$ I is its power set, or set of subsets. $\mathcal{P}\left(\{S_w^p\}_j\right)$ is therefore the power set of some subset of size $j$. Note that for any set of size $j$, its power set is of size $2^j$. For example, in traditional set theory, the set $\{1, 2\}$ has power set

$$\mathcal{P}\left(\{1, 2\}\right) = \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$$

with size $\left|\mathcal{P}\left(\{1, 2\}\right)\right| = 2^2 = 4$, since the power set contains both the empty set $\varnothing$ and the whole set $\{1, 2\}$ as elements. The inclusion of the empty set is required for logical consistency in the standard formulation of set theory.

## 5. Nodes as Perspectives (P)

In DSRP Theory, any node can also be a point-of-view. The Perspectives Rule or "P-Rule" provides that *perspectives are universally structured as co-implying elements: point ($\rho$) and view ($v$)*. This means that (as shown in Figure 9) any of the $n$ nodes in a network has the potential to be a point (the vantage point from which looking/framing occurs) or a view or part of a view (that which is being framed or observed).

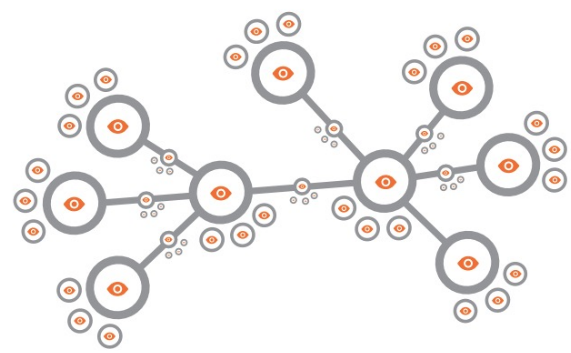


**Fig. 9.** Point-View Perspectives in the Network

The formula for calculating the degrees of freedom for point-view perspectives in $n^2$ because for every $n$ there exists a point and for every point there exists a view. The view can of course be an individual node, a system of nodes, a system of nodes and relationships, etc.

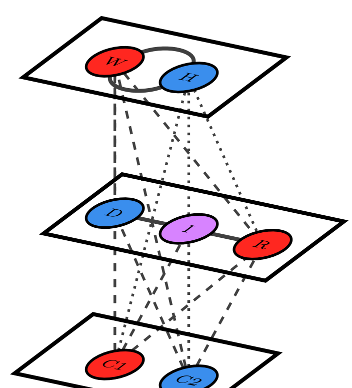



**Fig. 10.** Perspective graphs

*To adjust these formulae for situations in which one wishes to include the "self-part" structure, one more part for each class of subgroupings, corresponding to the "self-part" for each subgroup is included. Because all wholes are by definition a part that make up 100% of the whole, it is sometimes important to account for self-as-part. For example, for the $\{123\}$ root system, $\{123\}$ as the self-part at the $\{123\}$ substage, and $\{12\}$, $\{13\}$, $\{23\}$ as the self-parts at the other substage must be included. This gives an additional $1 + 1 * 3 = 4$ parts. In general, at root stage $n$, there is one additional part for each subgrouping, which is an additional $\sum_{j=2}^{n} \binom{n}{j}$ parts

From our discussion thus far we get that the degrees of freedom $\mathbb{F}$ in any network is:

$$\mathbb{F} = \sum n + n_p = n^2 + 2(n^2) + (2^n - 1) + n^2$$

## Heuristic to Determine Potential Complexity of P

The potential complexity of P at an arbitrary root stage is defined as the maximum number of points $p$ and views $v$ that can be formed from $n$ root identities, without compounding with D, S, or P. The P-counting is identical to the R-counting performed above. Again, the first count does not include "self-perspectives," and the formulae are adjusted later.

Each root identity forms the point of a perspective and the view of another perspective. When "self-perspectives" are not included, the case of one root identity is again trivial, and has zero degrees of freedom. In the case of two root identities for example, $\{1\ 2\}$, the possible perspectives are $1 \xrightarrow{p\ \ v} 2$ and $1 \xleftarrow{v\ \ p} 2$. As with the R-counting, these perspectives can be enumerated by the orderings of the identities: $1 \xrightarrow{p\ \ v} 2 \cong 1 \lessdot 2$, and $1 \xleftarrow{v\ \ p} 2 \cong 2 \lessdot 1$. In the case of three root identities $\{1\ 2\ 3\}$, the possible perspectives are

$$\begin{matrix} 1 \lessdot 2 & 1 \lessdot 3 & 2 \lessdot 3 \\ 2 \lessdot 1 & 3 \lessdot 1 & 3 \lessdot 2 \end{matrix}$$

And as with the R-counting, each of these P's contains two degrees of freedom, one for each element. So for $n \geq 2$, the maximal number of $p$'s and $v$'s is

$$\#p + \#v = 2 \times \frac{n!}{(n-2)!} = 2n\,(n-1)$$

Once again, perspectives on subgroupings such as $1 \lessdot \{2\ 3\}$ are not counted, since the description of such perspectives refers to compound DSRP structure, such as the composition of S and D with P. As with the R count, to correct these formulae for the inclusion of "self-perspectives" such as $1 \lessdot 1$, one additional point and one additional view for each possible self-perspective must be added. This gives an additional $2n$ elements, bringing the total potential complexity to $2n^2$. Note that for the trivial case of one root identity, this count gives two degrees of freedom, corresponding to the one point and one view of the self-perspective.

$$\begin{matrix} 1 \lessdot 2 & 1 \lessdot 3 & 2 \lessdot 3 \\ 2 \lessdot 1 & 3 \lessdot 1 & 3 \lessdot 2 \end{matrix}$$

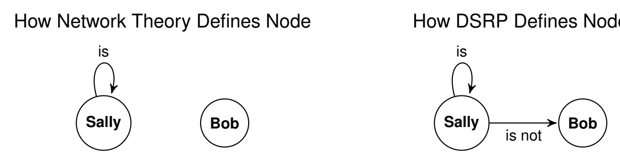
For simplicity sake we can use a basic heuristic that for any system of $n$ things, there are a possible $n$ points seeing $n^2$ views, which we can express as $n \lessdot n^2$. Thus for a system where n is equal to 2, 3, or 4 respectively, the table below with nodes A,B,C,D illustrates the heuristic for Perspective:

| n | Perspective Combinations | | | | Heuristic |
|---|---|---|---|---|---|
| 2 | $A \lessdot A$ | $A \lessdot B$ | | | $n \lessdot n^2 = 2 \lessdot (2*2) = 2 \lessdot 4$ (i.e., 2 points see 4 views) |
| | $B \lessdot B$ | $B \lessdot A$ | | | |
| 3 | $A \lessdot A$ | $A \lessdot B$ | $A \lessdot C$ | | |
| | $B \lessdot B$ | $B \lessdot A$ | $B \lessdot C$ | | $n \lessdot n^2 = 3 \lessdot (3*3) = 3 \lessdot 9$ (i.e., 3 points see 9 views) |
| | $C \lessdot C$ | $C \lessdot A$ | $C \lessdot B$ | | |
| 4 | $A \lessdot A$ | $A \lessdot B$ | $A \lessdot C$ | $A \lessdot D$ | |
| | $B \lessdot B$ | $B \lessdot A$ | $B \lessdot C$ | $B \lessdot D$ | $n \lessdot n^2 = 4 \lessdot (4*4) = 4 \lessdot 16$ (i.e., 4 points see 16 views) |
| | $C \lessdot C$ | $C \lessdot A$ | $C \lessdot B$ | $C \lessdot D$ | |
| | $D \lessdot D$ | $D \lessdot A$ | $D \lessdot B$ | $D \lessdot C$ | |

## 6. Distinguishing Nodes (D)

Now that we have defined the S, R, and P Rules, we can better answer the seemingly simply, but surprisingly complex, question, *what defines a node?* In network theory, an abstract thing called a "node" is defined by assigning it an "*Id*" or "*identity*" (i.e., a letter, number, symbol, or data, etc.). Thus, the way a node is distinguished, and subsequently defined, is given by its *identity*. Let's say for example that the node is labelled "$A$." We therefore can call that node "$A$" and say that its identity is "$A$" or alternatively, the "node $A$." Let's say the node's identity is "Sally":

**Table 6. Distinguishing a Node**




In actuality, "Sally" is merely the label for the node and its identity is far more complex. The identity of any given node is established by a complex formula of not only what it is, but also all of the things in its universe that it *is not.* This is sometimes thought of as qualitative and "contextual" but it is quantifiable. The Distinction Rule of DSRP (or "D-Rule") states that *Distinctions (D) are universally structured as co-implying elements: identity (i) and other (o).* This means that in a network comprised of nodes $A$, $B$, $C$, and $D$, as in the Konigsberg example, the identity of A is not merely A alone. The identity of $A$ also includes the following (left column of Table 7):

**Table 7. The identity of A**

| | |
|---|---|
| $A = A$ | $A$ **is** $A$ |
| $A = \neg B$ | $A$ **is** not-$B$ |
| $A = \neg C$ | $A$ **is** not-$C$ |
| $A = \neg D$ | $A$ **is** not-$D$ |
| $A = \neg(BCD)$ | $A$ **is** not-$(BCD)$ |

The left column in Table 7 are notations that are explicated in the column on the right. The right column in Table 7 can be read as *existential* statements of the identity of $A$. By existential, we mean that something *is* (from the verb "to be"). Note that all of the statements about $A$ are *is*-statements; whether they refer to characteristics that A *is* or those that A *is* not. Thus identity of $A$ is a listing of all the things $A$ is, which includes what it is, not. In other words, these are all the things that A *is* and A *is* not—the things about $A$ and the *other* things that help to collectively form $A$'s *identity* (See Figure 11 for the visual version of this idea).

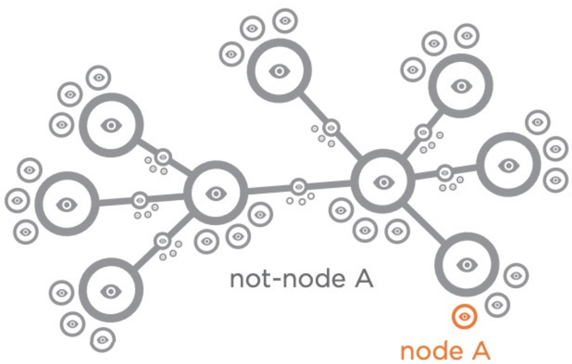
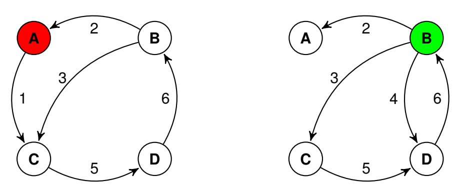



**Fig. 11.** Identity-Other Distinctions in Networks

Earlier, in Table 7, the definition of $A$ not only accounts for what $A$ *is* but also what $A$ *is not*. Due to our discussion thus far about the potential complexity of not only D, but also S, R, and P, it becomes clear that the variables associated with identity and other for any given identity are not two-fold but many-fold and are therefore more numerous. Let us look at a system of 4 interrelated nodes from two different perspectives, that of Node $A$ and Node $B$ noting the single difference in their perspectives being the relationship between $A$ and $C$:

**Table 8. 2 Perspectival Networks**




Network from A's perspective Network from B's perspective

Where A recognizes that a relationship exists between A (itself) and C, B does not recognize this relationship. Hypothetically, if this was all the information we had about this network then we could only conclude that A *is* both *related to* and *not related* to C depending on which perspective is to be believed. This is not as abstract as it sounds, consider, for example the recent reports about a virus (COVID-19) whose identity continues to be an enigma to us; a recent research report shows that it is asymptomatically contagious while another research report shows that it is not. ***Is*** COVID-19 asymptomatically contagious? It depends who you ask. If you take the total information available, then the answer is *yes* and *no* (or *maybe* or *we don't know*). In the past twenty years Pluto has changed from being a planet to not being a planet to once again being a planet. ***Is*** Pluto a planet? It depends who you ask. If you take the total information available, then the answer is *yes* and *no* (or *maybe* or *we don't know*). The deeper meaning is that even something as simple as defining A is probabilistic in nature. In other words, the definition of A is a probability cloud of its DSRP.

Note again the existential nature of the term ***is*** (from the verb *to be*). In order to define any node A, we must define what it *is*. We must therefore define what is sometimes call, vaguely, the "context" of A. But this context is quantifiable and important, as it not only influences A and vice versa, it also *defines* A. What we have been describing thus far is the universal structures that elucidate and quantify this "context." Table 9 builds on the perspectival networks in Table 8 to further define A. Not how different Table 7) is from Table 9.

**Table 9. A is all of these things...**

| | | |
|---|---|---|
| $A$ | ***is*** | $A$ |
| $A$ | ***is*** | related to $B$ |
| $A$ | ***is*** | related to $C$ (according to $A$) |
| $A$ | ***is*** | related to $D$ |
| $A$ | ***is*** | part of($ABCD$) |
| $A$ | ***is*** | comprised of [unknown] parts |
| $A$ | ***is*** | a relationship between $B$ and $C$ (according to $A$) |
| | | |
| $A$ | ***is*** | not-$B$ |
| $A$ | ***is*** | not-$C$ |
| $A$ | ***is*** | not-$D$ |
| $A$ | ***is*** | not-($BCD$) |
| $A$ | ***is*** | *not* related to $C$ (according to $B$) |
| $A$ | ***is*** | not a relationship between $B$ and $C$ (according to $B$) |

What Table 9 illustrates is that the existential nature of A (i.e., its identity) is defined not merely by what it is and what it is not as described in Table 7, but also by what it is and is not related to, what it is and is not part of or a whole for, and all of these conditions according to various perspectives. In other words, the very identity of any node, is a part-whole system of "definitions" distinguishing all the things that it is and also all the things it is not.

## Heuristic to Determine Potential Complexity of D

Utilizing the D-Rule, we can count the number of *other* ($o$) variables as a single set or part-whole system we can use the formula $2n$ in any network of $n$ nodes. This is because for every node (i.e., for every *identity* ($i$) variable) there is an *other* ($o$) variable. But, we will see that both the *identity* and *other* variables are not a single thing, but a collection of things (a.k.a., a part-whole system).

## 7. Conclusion

Summarizing these heuristics[†] for D, S, R, and P we find that the following can be used in most cases:

- Distinctions can be quickly calculated using the formula $2n$
- Systems can be quickly calculated using the formula $2^n - 1$
- Relationships can be quickly calculated using the formula $n(n-1)/2$ for single Relations; $n(n-1)$ for two-way Relations; $n^2$ for single relations included self-relations, and; $2n^2$ for action-reaction variables on two-way relations including self-relations.
- Perspectives can be quickly calculated using $n \lessdot n^2$, where there are $n$ points seeing $n^2$ views.

Network Theory offers significant insight into the structural properties of complex adaptive systems that allow us to see the full potential of the complexity of both the real world and one's thoughts about it. DSRP Theory provides a universal cognitive grammar (UCG) and numerous derivative heuristics that help count the maximum possibilities in the structure of any network—providing a short cut and maximum efficiency in understanding complex networks, including human thought. This universal cognitive grammar and these heuristics also provide the basis for making structural predictions about what might be, what is not yet known, or what has yet to be discovered.

[†] The counting of the maximal number of elements for D, S, R, and P in terms of $n$ root identities serves as a heuristic measure for the potential DSRP complexity of a given mental model. However, this type of counting may not represent a meaningful measure of the "useful" complexity of a given mental model, or the likely complexity of a typical mental model. More specifically, this counting of degrees of freedom in terms of indivisible root identities introduces an inherent bias into the counting procedure, which destroys some of the symmetry of DSRP. Given the importance of the elementary process of splitting a "primitive" identity into further parts, and the automatic and essential character of other compound DSRP operations, it seems that this method of quantifying DSRP does not provide the most natural way of thinking about DSRP dynamics, even if it may be used to correctly enumerate the total number of possible elements. Some more detailed comments about these caveats are in order. 1. Counting in terms of the number of "root identities" introduces "identity bias" in the framing of the complexity count. In some situations, it may be advantageous to perform other counts. For example, a DSRP user may be interested in counting the maximal number of part-wholes as a function of R's manifested in a given mental model, instead of counting in terms of $i$'s. 2. Treating root identities as indivisible introduces an additional bias into the count. It is a natural consequence of S that any whole may in principle be decomposed into parts. However the counting above regards the root identities as primitive and indivisible. It accounts for the additional degrees of freedom that describe root identity part structure by partitioning them within the root stage that has the correct total number of identities. This form of identity bias occurs because we are counting elements as a function of identities. One of its consequences is that certain natural DSRP co-implications, such as the co-existence of parts and their identities, are not manifest in the counting protocol at each root stage. 3. This identity bias also influences other natural co-implications and compound DSRP operations. For example, in the S-count for {1 2 3}, we were compelled to allow wholes such as {1 2}. However by pattern co-implication, it is automatic that such a whole has an identity for example, and therefore the total number of identities present is increased. But these new identities are ignored by the count: it counts only parts and wholes as a function of the primitive identities. It regards these new identities as a compound DSRP operation, in particular a composition of D with S, to be accounted for at a later stage with more identities. Moreover, such compound operations could continue ad infinitum: the subsystem {1 2} is automatically related to {3}, and has a perspective on {1 2 3}, etc. So this truncation of the most basic co-implications, in order to organize the count by root identities, biases the complexity measure in ways that are unnatural for some applications, even though these degrees of freedom can be accounted for step-by-step by adding more root identities. 4. Finally, given the compounding nature of DSRP described in 3, the maximal complexity of a possible mental model is perhaps unbounded. However the "useful" complexity of a mental model is clearly bounded by both the saliency of the degrees of freedom and the cognitive tendencies of a typical thinker to suitably coarse-grain. In relation to the counting procedure given above, it should be clear that simply adding the maximal number of elements of each pattern in terms of $n$ root identities will yield a very restricted notion of maximal complexity. As stated, the counting procedure is identity-biased and ignores all co-implications and compound operations. This complexity measure is then better described as "the maximal number of elementary degrees of freedom in the minimal identity representation of a mental model, as a function of number of elementary identities."

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